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A COMPARISON OF VECTOR AND NETWORK-BASED MEASURES FOR ASSESSING DESIGN SIMILARITY

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ABSTRACT

In order to retrieve analogous designs for design-by-analogy, computational systems require the calculation of similarity between the target design and a repository of source designs. Representing designs as functional abstractions can support designers in practicing design-by-analogy by minimizing fixation on surface-level similarities. In addition, when a design is represented by a functional model using a function-flow format, many measures are available to determine functional similarity. In most current function-based design-by-analogy systems, the functions are represented as vectors and measures like cosine similarity are used to retrieve analogous designs. However, it is hypothesized that changing the similarity measure can significantly change the examples that are retrieved. In this paper, several similarity measures are empirically tested across a set of functional models of energy harvesting products. In addition, the paper explores representing the functional models as networks to find functionally similar designs using graph similarity measures. Surprisingly, the types of designs that are considered similar by vector-based and one of the graph similarity measures are found to vary significantly. Even among a set of functional models that share known similar technology, the different measures find inconsistent degrees of similarity – some measures find the set of models to be very similar and some find them to be very dissimilar. The findings have implications on the choice of similarity metric and its effect on finding analogous designs that, in this case, have similar pairs of functions and flows in their functional models. Since literature has shown that the types of designs presented can impact their effectiveness in aiding the design process, this work intends to spur further consideration of the impact of using different similarity measures when assessing design similarity computationally.

1 Introduction

As information across different domains of designs becomes increasingly accessible, it has become possible to leverage this data to provide sources of inspiration to designers. Design-by-analogy is a method designers have used to transfer knowledge from cross-domain sources and apply it to a target domain [1, 2]. When humans retrieve analogies on their own, they can have difficulty moving past surface-level similarities, often finding within-domain analogs that share readily-observed external attributes rather than underlying structural similarity. Computational systems grant the opportunity to search for analogies within a larger space and automatically determine the relevant analogous design. Since these systems do not rely on surface-level similarities, they are able to retrieve more distant analogies based upon underlying functional patterns across domains [3]. The presentation of computationally-determined real-time analogical stimuli during early-stage design has been found to help designers produce novel outcomes [4].

Significant work has been done on analogy retrieval based on semantic representations of products such as design descrip-

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tions from a design problem solving session [4] or crowd-sourced design schema representations [3]. However, it can be advantageous to focus on functional analogies to further remove possibilities for fixation on surface similarities. To incorporate the advantages of functional representations, researchers have also developed a method to retrieve analogies using a function-based approach on semantic data [5,6]. However, functional models offer an alternative to semantic descriptions, providing structured system or subsystem level representations that are useful for designers [7]. In addition, functional models have been useful in cases such as bio-inspired design, where the source vocabulary is significantly different from that of the target [8]. A significant benefit of functional models is their ability to be mapped to a mathematical space, where a variety of measures are available to characterize the distance between them.

Design cognition work has shown that different analogical distances can impact a designer's ideation processes, even on a neural level [9]. It can be desirable to systematically control for distance in computational design by analogy in order to leverage the effects of near vs. far analogs. In that case, it is critical to clearly define near and far through a measure of similarity. However, it is possible that using different measures can return drastically different analogs and that different measures are appropriate for different contexts. This work empirically questions the meaning of *similarity* in engineering design by explicitly comparing multiple similarity measures and how they measure similarity across functional models. Specifically, we investigate the identification of similar functional models using vector space based methods, such as those already used frequently within the engineering design community. In addition, we explore the possibility of representing functional models as networks and applying graph similarity measures. The work has implications for defining analogical distance for computational systems, but is applicable to any context where it is necessary to systematically determine the similarity between designs.

1.1 Similarity in Engineering Design

Similarity has been addressed in the engineering design community through the development of computational systems for retrieving analogous designs and through assessing design similarity in ideation. A variety of metrics have been used to define the similarity between designs. For instance, critical function chains have been extracted from functional models and a variety of matching metrics have been applied to determine similarity between the chains to find analogies [10]. A similarity metric has also been developed to compare functional models from a product repository using customer needs [11]. The metric has been shown to successfully find relevant analogies as demonstrated by its example application in finding analogies to drive the design of a new guitar pickup winder [11]. Since it can be difficult to ensure that domain specific knowledge like customer needs is encoded in functional models, later work has used the metric directly without customer need weightings, [12, 13] demonstrating an underlying assumption that notions of similarity will not significantly change without that information. Additionally, a significant body of work has used natural language processing in order to retrieve functionally similar analogies from existing repositories (e.g., the U.S. patent database). In this case, as it is necessary to compare texts of different lengths, the cosine similarity measure is typically used [5, 6]. The use of latent semantic indexing and cosine similarity has been compared to the quantitative metric for similarity based on customer needs in the context of quantifying similarity between automatically generated concepts [14]. Finally, recent work has applied KL-divergence to determine similarity between designs in a way that embodies known principles of similarity from cognitive science. This similarity metric models knowledge and maps design characteristics to performance measures before using KL-divergence. The metric has been utilized to measure similarity between design problems [15].

The engineering design community has also previously reviewed the use of similarity measures to compare designs at different stages of the design process. In a survey of similarity metrics used in engineering design, spatial function, vector space, edit distance, template model, and information theory approaches were evaluated holistically but qualitatively (i.e., no empirical results). It was determined that an edit distance or information theory approach should be most suitable to compute similarity between designs at the function structure stage of the design process [16]. While various types of metrics have been used to try to find design similarity, there has been a lack of empirical testing on how the measures directly compare to each other and therefore, no way to determine the appropriate occasion to use each one.

Quantitative measures of similarity between different designs are important given that prior research in engineering design has revealed that analogies of varying distances can have different impacts on design outcomes. Analogical distance refers to how close the source design is to the target design. The analogies have been divided into near-field and far-field analogies. Previous work classifies within-domain systems as near and out-of-domain systems as far. Significant work has also been done to determine if the analogical distance affects the novelty or quality of ideas since far-field ideas may have functional similarities that make them transferable. This has revealed contradictory results and has indicated that there is a problem of "too far" in analogical distance [5]. At the same time, the results of this body of work indicates that the types of analogous designs presented affect their usefulness to designers. If the similarity measure significantly influences what is considered functionally similar, then different similarity measures have the potential to offer new ways to find near-field and far-field analogies for design, or alternatively, necessitate reassessment of the importance in the choice of metric used to find analogous designs.

1.2 Functional Modeling

Functional modeling is a group of methods by which a product can be decomposed into its key functions, providing an abstraction of the system that is useful for various stages of the design process. In early-stage design and concept generation, functional modeling can be used to decompose a complex problem into simpler sub-problems by using a black box approach [17]. In addition, functional modeling provides a way to capture important knowledge about existing designs that is not captured in traditional documentation such as CAD models [18]. There are limitations to using functional models, especially in early-stage design, since existing functional models are created through reverse engineering. Additionally, there have been several approaches to functional modeling [17, 19–22], but the development of a functional basis has provided a common design language, allowing meaningful comparisons at the functional level [23].

The functional basis allows a product to be represented by labeling its functions and flows in pairs using a standardized vocabulary. There are three primary classes of flows (material, energy, and signal) and eight primary classes of functions (channel, support, connect, branch, provision, control magnitude, convert, and signal). These function and flow classes can have a further secondary and tertiary specification, maintaining flexibility in the level of abstraction in which a system can be modeled [18,23–25]. Once a product is modeled using these function-flow pairs, the model can be mapped to a variety of mathematical representations that can be used for further analysis. Specifically, the functional models can be mapped to a vector space or network / graph.

1.3 Measures for a Vector-Space Representation

The majority of prior work utilizing functional models has mapped the functional models into a vector space for similarity analysis. A functional model can be mapped to a vector space by building a binary vector from each of the function-flow pairs. For example, this mapping was used for the quantitative metric for similarity based on customer needs, as well as to investigate the effect of varying the level of abstraction on functional similarity [11, 12]. A higher level of abstraction has been used where, instead of a binary vector of function-flow pairs, a smaller vector is built using only functions and flows separately [13].

In the method developed by McAdams and Wood, the elements of each functional model mapped vector is weighted according to a customer needs rating. Then, product vectors are constructed into a product-function matrix, which is normalized for product complexity and customer enthusiasm rating, and renormalized to unity. Finally, the inner product is calculated between each product vector [11]. When the same process is followed without assigning weightings according to customer needs, the results are equivalent to applying the cosine similarity metric, which measures the cosine of the angle between the two non-zero vectors. The cosine similarity varies between zero and one, with one being perfect similarity. The cosine similarity has commonly been used in an engineering design context because of its applicability to compare different lengths of text for semantic similarity. However, in the absence of domain-specific weighting on specific functions, other metrics are available to quantify the similarity or distance between two binary vectors and may be applicable to compare functional models. Many metrics have been developed for this comparison according to different needs, only some of which are investigated here [26].

1.4 Measures for a Network Representation

Networks mathematically represent the connections between entities. A network consists of vertices (or nodes) that are connected by edges. They have been widely used in applications such as social network analysis and have recently been utilized by the engineering design field. For example, recent work has used networks to find bridges between ideas from different domains using topic models [27] and represent a conceptual design space for early-stage design [28]. In both cases, the networks are built from information in text documents and are not specific to functional representations. In addition, networks have been used to represent complex systems and model system failure [29, 30]. Network structures have been used for bio-inspired design of a power network [31] and to represent influential function models in a product architecture [32]. Functional models have been previously represented as networks to investigate product transformation using graph edit distance [33]. The edit distance has also been previously noted as a relevant similarity measure for comparing function structures [16].

Just like there are a variety of vector-based approaches, there are several additional network similarity measures that can capture the structure of a network. These can be divided into ones that require known node-correspondence – having a set or subset of matching nodes – and those that can have unknown node-correspondence [34]. In addition, network similarity measures can rely on certain network properties including if the networks are undirected or directed (pointing only in one direction) as well as if they are unweighted or weighted (edges have a positive continuous value) [35]. As such, there is the potential to represent the functional model as a network in several ways.

2 Research Methodology

This paper investigates the impact of using different similarity measures to discover functional analogies from a product repository. In comparing the different measures, we intend to gain insight into the meaning of similarity in engineering design. First, the functional models were mapped to the desired mathematical space (a binary vector or a network) as shown in Figure 1. Then, a similarity matrix between all of the models in the repository was computed for each measure. The similarity matrices were all range normalized so the similarity score was between zero and one. The measures were quantitatively and qualitatively compared to gain insights regarding how each measure influences which designs are considered similar.

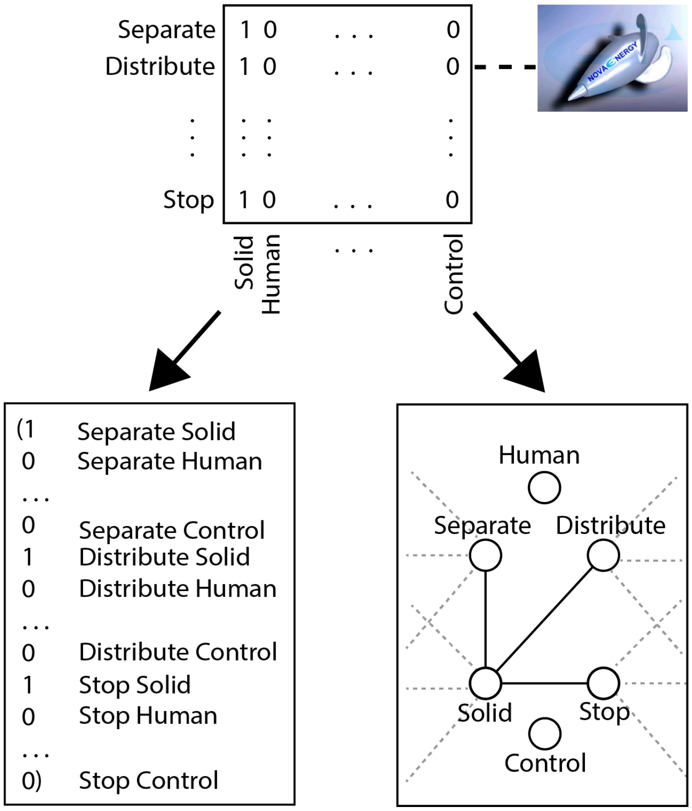

FIGURE 1. The functional model data is in the form of a binary matrix connecting functions and flows. This matrix is flattened to a vector (left) or used as an adjacency matrix and represented as a network (right) as shown in this example with the Nova Energy Tuna Turbine. If the matrix contains a 1, an edge connects the function node to the flow node. If the matrix contains a 0, those nodes are not connected. Functions can only be connected directly to flows, not other functions, and flows can only be connected directly to functions, not other flows.

2.1 Functional Model Data

The functional models used in this work came from previous work where functional models were developed for 39 energy harvesting devices using the functional basis [13]. This data was analyzed since the set of energy harvesting devices is inherently similar by technology. As a result, the similarity measures were expected to find systems that share a similar working principle to be "near". The systems included energy harvesters of different types and ranged from prototypes to commercial products. The energy harvesting functional models were categorized into technological sub-domains as follows:

9 inductive vibration harvesters
6 piezoelectric vibration harvesters
6 wind harvesters
3 ocean-current/wave harvesters
6 solar harvesters
5 thermal harvesters
4 hybrid harvesters

Details about the categorization of the energy harvesting devices can be found in Appendix A. The functional models of these energy harvesting devices were developed using 21 functions and 16 flows (a list of these can be found in Appendix B). The functions were specified to the secondary level while the flows are sometimes specified to the tertiary level. For example, the flow *mechanical energy* was clarified further to be *rotational* or *translational*. The functional models did not include information about the sequencing of function-flow pairs in the system, the repetition of any functions or flows, or the relative importance of any functions/flows.

2.2 Measures of Similarity Using Vectors

The functional models were mapped to a vector space by building a binary vector from the existence of function-flow pairs in the system. Therefore, for n functions and m flows, each functional model was represented by a vector of zeros and ones of length $n \times m$. These vectors were then used for any similarity computation. The similarity measures were chosen only if they were applicable to binary data. The similarity measures do not always satisfy the specific definition of a *metric* and therefore are not referred to as such. In addition, there was an effort only to compare measures that have been previously used in the engineering design field. It should be noted that some measures are referred to (or calculated) as distances and dissimilarities. These were always converted to measures of similarity before comparison. The vector-based similarity measures explored in this work are described in more detail below.

Simple matching coefficient (SMC). The Hamming distance is the number of differences in corresponding positions of two binary vectors. Eq. 1a shows that the formula for Hamming distance is

$$Hamming\ distance = \sum |x_1 - x_2|, \tag{1a}$$

where x_1 and x_2 are the two binary vectors being compared. The measure is often divided by n (vector length) in computational packages to obtain a proportion. This proportion can then be converted to the simple matching coefficient (*SMC*) as

$$SMC = 1 - \frac{\sum |x_1 - x_2|}{n}. \tag{1b}$$

The SMC can only be used on binary data and is useful if the features are symmetric. This means that the absence or presence of the feature carries equal information.

Jaccard similarity coefficient. The Jaccard similarity coefficient and the SMC are close in their comparison of binary vectors. Eq. 2 shows that the formulation of the Jaccard similarity coefficient is

$$Jaccard\ similarity = \frac{|x_1 \cap x_2|}{|x_1 \cup x_2|}, \tag{2}$$

where x_1 when x_2 are the two binary vectors being compared. Unlike the SMC, however, the Jaccard similarity coefficient excludes any features that are not present in either vector. Therefore, it only accounts for mutually present matches between the vectors. The Jaccard similarity as defined above can be used on binary data and modifications allow the measure to be used with weights or probability distributions.

Cosine similarity. The cosine similarity determines similarity based on the the angle between two vectors in a vector space. It is

$$Cosine\ similarity = \frac{x_1 \cdot x_2}{||x_1||||x_2||}, \tag{3a}$$

where x_1 and x_2 are the two vectors being compared. The quantitative similarity metric developed by McAdams reduces to this measure when information about customer needs is not available. When used on binary data, the equation can be rewritten as

$$Cosine\ similarity = \frac{|x_1 \cap x_2|}{||x_1||||x_2||}. \tag{3b}$$

The numerator is the same as in the Jaccard similarity coefficient from Eq. 2. The cosine similarity can be used on binary data but does not need the data to be binary. It is commonly used in the context of comparing text documents of different lengths since it compares the orientation of two vectors in a high-dimensional abstract space.

2.3 Measures of Similarity Using Networks

To represent a functional model as a network, the functions and flows of each product were first mapped to a binary matrix. Each function and flow was represented as a node and the edges were determined by the values in the matrix. Edges between functions and flows existed only if the binary matrix had a 1 in the corresponding row and column. The network comparison measures were chosen so that they would be applicable to undirected and unweighted networks, as the functional models do not contain information about the direction of connections between different functions or about relative importance of functions (note that some graph-based similarity measures are specifically developed to handle these properties). In addition, feature-based approaches for network comparisons (clustering coefficient, centrality, etc.) were not considered. The graph similarity measures were formulated as distances and then converted to similarity for comparison. The networks were visualized and analyzed using the NetworkX [36] and NetComp [37] libraries in Python.

Graph edit distance (GED). The graph edit distance has many implementations depending on the graph type. It refers to the required deletions, insertions, or substitutions of vertices or edges to make the two graphs isomorphic. In the case where the edges between the nodes depend only on the existence of a connection, the graphs can be compared using their adjacency matrices (A_1, A_2). The graph edit distance is then

$$GED = \frac{\sum |A_1 - A_2|}{2}. \tag{4}$$

The GED is useful if the matrix used (in this case, the adjacency matrix) contains useful information about the graph structure. Since it focuses on edge changes, the GED can be good for detecting local structure. In this implementation, the formulation is similar to the SMC (Eq. 1b).

Spectral distance. Spectral distances are based on the eigenvalues of a matrix. In this case, the spectral distance is defined as

$$Spectral\ distance = ||\lambda_{\mathscr{L}_1} - \lambda_{\mathscr{L}_2}||, \tag{5a}$$

where $\lambda_{\mathscr{L}_1}$ and $\lambda_{\mathscr{L}_2}$ are the eigenvalues of the Laplacian matrices ($\mathscr{L}_1, \mathscr{L}_2$). The Laplacian matrix is

$$\mathscr{L}_i = D_i - A_i. \tag{5b}$$

In addition to using the adjacency matrices (A_1, A_2) that are used in calculating GED, the spectral distance accounts for the degree matrices (D_1, D_2) through the Laplacian. The degree matrix is a diagonal matrix that indicates how many other nodes each node is connected to.

When the Laplacian matrix is normalized, the spectral distance can be used to compare graphs of different sizes. In addition, it does not require the nodes of the two graphs to be the same. When computing a spectral distance, the number of eigenvalues that are considered can be adjusted, allowing flexibility in considering community structure (fewer eigenvalues) or including local structure (more eigenvalues). Comparisons of several types of real world networks finds that spectral distance is a reliable measure for different applications [37].

DeltaCon distance. The DeltaCon distance is a graph comparison measure intended to account for the similarities in connectivity between two graphs. To do this, the pairwise node affinities are calculated for each graph and then compared to each other. The node affinities are calculated using a concept called fast belief propagation (FBP), an approximation of the loopy belief propagation algorithm. This is a message-passing algorithm often used on graphs in computer science [38]. The FBP matrix is

$$S = [I + \varepsilon^2 D - \varepsilon A]^{-1}, \tag{6a}$$

where ε is

$$\varepsilon = \frac{1}{1+d_{max}}. \tag{6b}$$

ε is the constant that accounts for the influence of neighboring nodes and is computed using the maximum value in the degree matrix (d_{max}). The FBP matrix can also be written as

$$S \approx I + \varepsilon A + \varepsilon^2(A^2 - D) + ..., \tag{6c}$$

demonstrating how it incorporates information about neighboring nodes using weighting. The final distance is then

$$DeltaCon\ distance = \sum |\sqrt{S_1} - \sqrt{S_2}|. \tag{6d}$$

Like the spectral distance, the DeltaCon distance uses both the adjacency matrix (A) and the degree matrix (D). Fast belief propagation is intended to track the spread of information through a graph, making the DeltaCon method good for local and global structure [37].

3 Results

The vector and network-based similarity measures outlined in Section 2 were used to find the similarity between the functional models of all pairs of devices in the energy harvesting data. The results were stored in similarity matrices and then analyzed with the objective of determining how the choice of similarity measure affects which functional models are considered similar to each other.

3.1 Overall Comparison of Similarity Measures

The similarity matrices, which are pairwise comparisons of the functional models as evaluated by each similarity measure, were plotted as a distribution of scores. The distributions illustrate the ability of the measures to distinguish functional models that share an inherent commonality (being from the same technology domain – in this case, energy harvesting devices). The kernel density estimate of each similarity measure is shown in Figure 2.

The mean and Pearson's coefficient of skewness of the distributions were calculated for each measure as shown in Table 1. A large negative coefficient of skewness indicates that the mass of the distribution is concentrated on the right (higher similarity), while a large positive coefficient of skewness indicates that the mass of the distribution is concentrated on the left (lower similarity).

The mean of the spectral similarity was the highest at 0.65 while the mean of the Jaccard similarity was the lowest at 0.31. The distributions of similarity scores from these two measures were highly skewed. However, they were skewed in opposite directions, which was unexpected. The Jaccard similarity distribution was concentrated towards lower similarity, while the spectral similarity was concentrated towards higher similarity. The distribution of the cosine similarity was moderately skewed. All other measures had distributions that had a low skew.

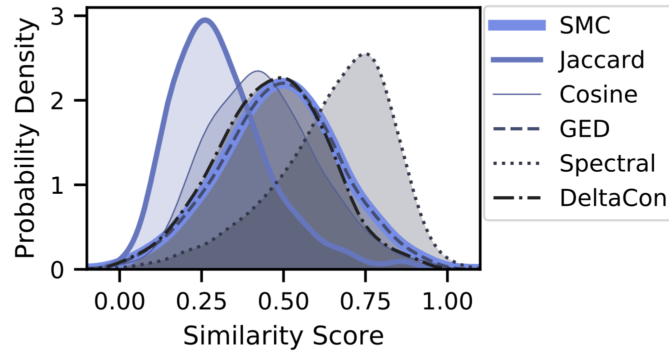

FIGURE 2. Distribution of normalized similarity measures.

TABLE 1. Mean similarity scores and coefficient of skewness of all energy harvesting devices. Shaded rows indicate measures that have highly skewed distributions.

Measure	Mean Similarity Score	Skew
SMC	0.49	-0.07
Jaccard	0.31	0.97
Cosine	0.44	0.35
GED	0.49	-0.07
Spectral	0.65	-0.81
DeltaCon	0.47	-0.06

Next, the similarity matrices were used to determine if the results returned by each similarity measure were distinct. For each energy harvesting system, every other energy harvesting system was ranked in order of its similarity to the initial system (tied rankings were included). The purpose of examining the rankings was to consider the possibility that even if the *value* of similarity between two measures was different, the relative *order* of systems returned may not differ much. These rankings were then analyzed using the Kendall rank correlation coefficient (Kendall's τ) to obtain a pairwise comparison between the methods. Due to existence of a distribution of rank coefficients that depended on the initial system and because of the small sample size, bootstrapping was used to find the 95% confidence interval for the pairwise rank coefficient, as shown in Figure 3. A positive rank correlation coefficient close to one indicates that the two measures being compared return rankings that are similar (i.e., they find the same types of functional models similar).

Despite there being a distribution of rank correlation coef-

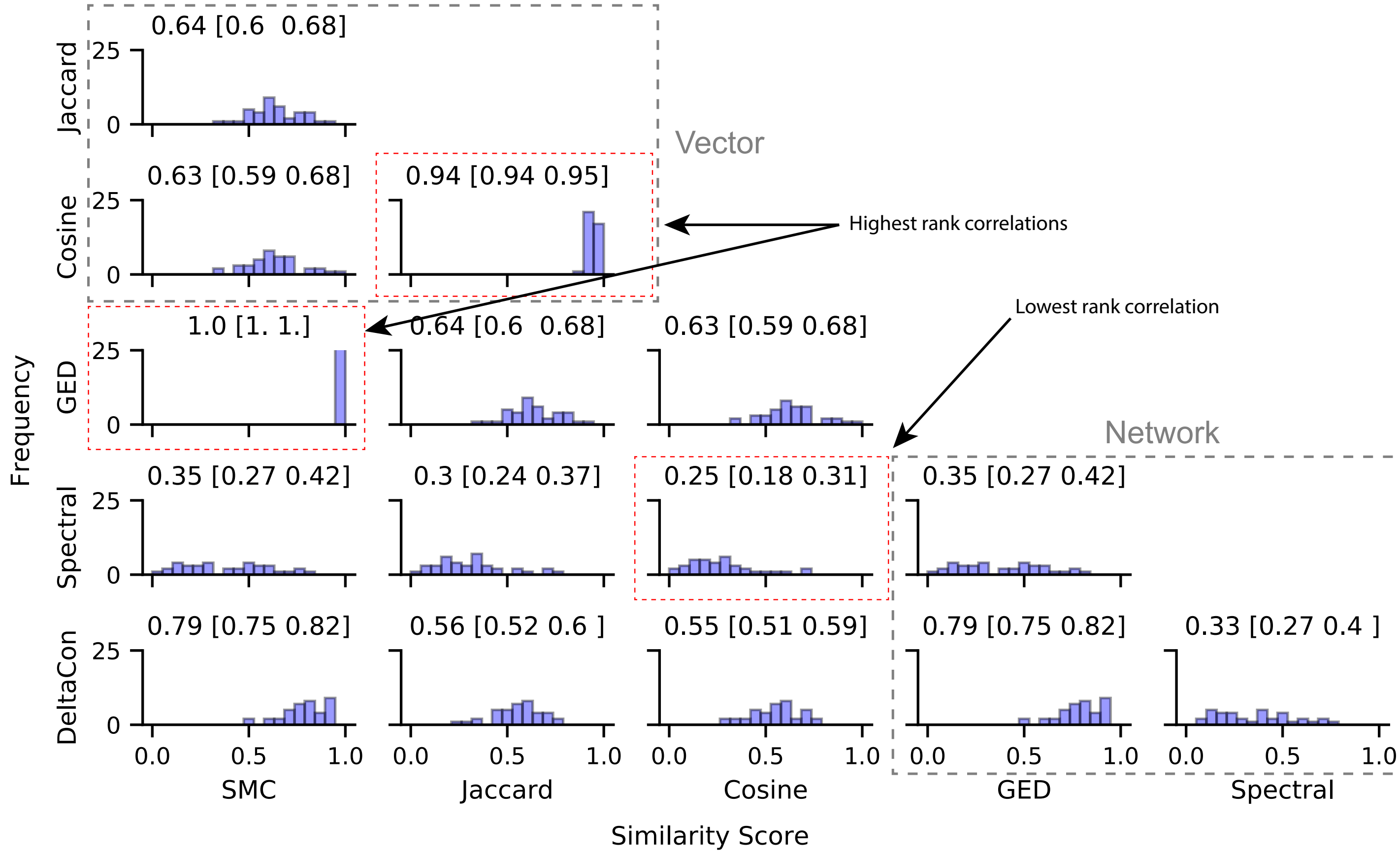

FIGURE 3. Kendall rank correlation coefficients between similarity measures are shown, using each of the 39 devices as the ”target” design. The mean and 95% confidence interval from bootstrapping (n=500) is shown for each pair of measures.

ficients, the rank correlation analysis revealed only a moderate correlation between most similarity measures. However, as expected from the mathematical formulation, the SMC and GED similarity measures returned the same results. The Jaccard and cosine similarity measures were highly correlated within a relatively narrow interval. In addition, the spectral similarity measure showed a very weak correlation with all vector methods and the least correlation with any other method overall, including the DeltaCon distance, the only other measure that included the use of a degree matrix.

3.2 Most/Least Similar Systems Across All Measures

The rankings used to find the rank correlation coefficient were used to analyze the results that were returned using each pair of similarity measures. The five systems ranked highest and lowest for a measure were compared with the five systems ranked highest and lowest for another measure. The systems that appeared in the top (or bottom) five for a pair of measures were counted. The counts for the intersection did not account for the order in which the systems appeared. Figure 4 shows a distribution of the intersection of the top/bottom results as a pairwise comparison of measures. The vector measures, in general, found similar results since they had four to five overlaps in most cases. Comparing the network measures revealed a larger spread of overlap, particularly with the spectral graph similarity measure. While the GED and DeltaCon measures led to similar results in the top five, the results from the spectral measure did not appear to overlap with the other network measures. There was a wider spread in the five *least* similar systems, even among the vector measures, unless the measures were highly correlated (e.g. Jaccard and cosine). Some measures had more overlap in the five systems considered the *most* similar than in the five systems considered the *least* similar, indicating the potential that different measures may be needed when looking for the two ends of the spectrum. The collection of five least similar devices represents systems that are the farthest within-domain systems. However, the categorization of “within-domain” was determined manually, and it is possible that the systems considered the least similar were functionally quite different from energy harvesting devices in general. Alternatively, it is possible that the measures agreed

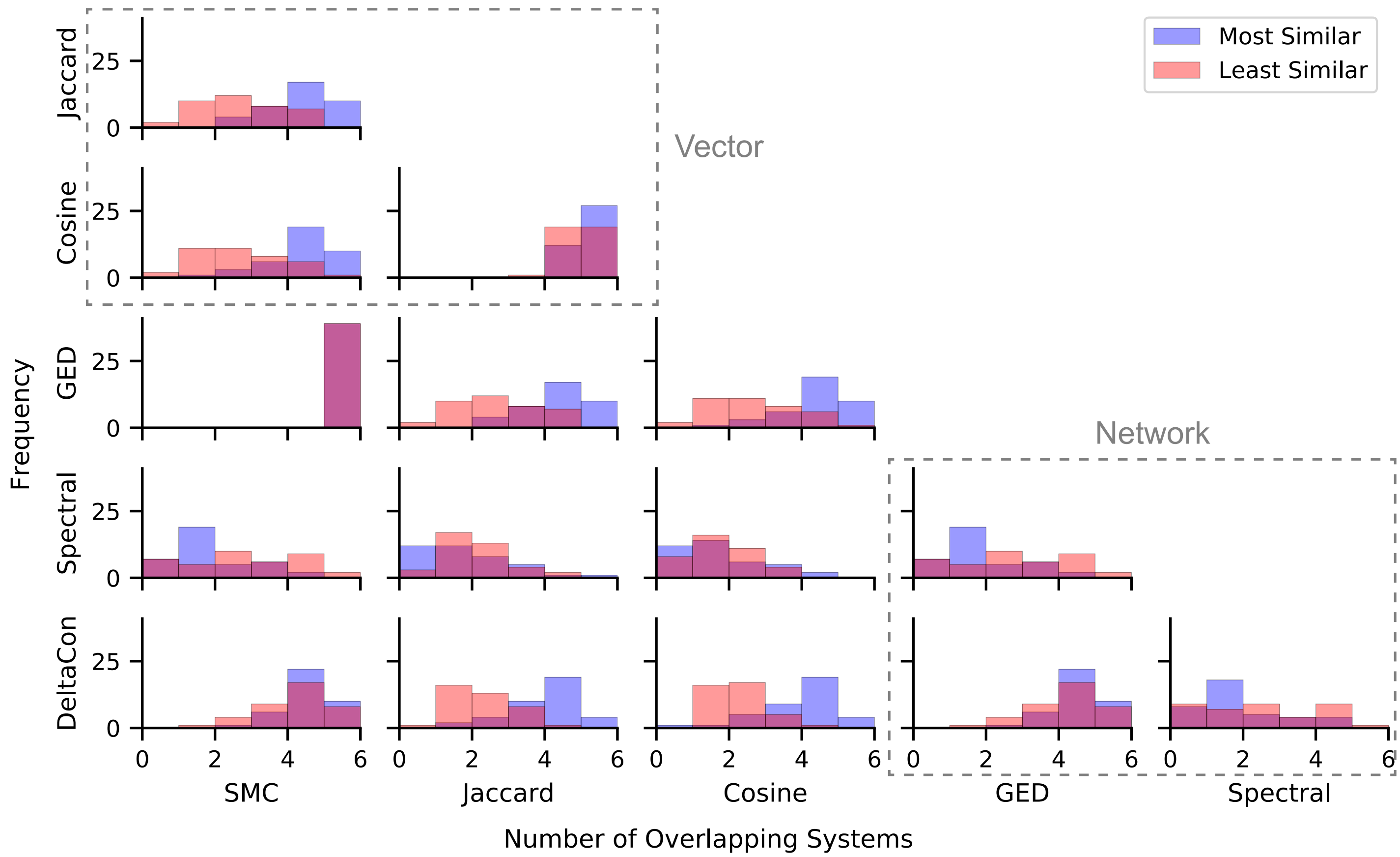

FIGURE 4. Pairwise comparison of similarity measures and the overlap in the five most/least similar systems returned for each system.

more on which systems are similar than on which systems are most dissimilar.

3.3 Comparison of Measures within Categories

The functional models all share the intended purpose of being energy harvesting devices, but each device was further labeled as a specific type of energy harvesting device (wind, solar, etc.). It was expected that devices within these categories would have similar working principles. The categorizations for each device can be found in Appendix A. The mean similarity was calculated for the systems within these predefined energy categories as shown in Table 2. Highlighted cells indicate category-level mean similarity scores that are not greater than or equal to the overall mean similarity score. The within-category similarity was generally higher than the mean similarity of all energy categories, although statistical significance was not determined due to the small sample size. Hybrid systems, which were predefined to contain multiple energy categories, were an exception and had a lower within-category similarity. In addition, piezoelectric devices had a lower within-category similarity using the Jaccard and cosine similarity measures, and solar devices had a lower within-category similarity using the spectral similarity.

Given the higher similarities for within-category devices, and that within-category devices should reasonably share the same working principle, it was expected that the device pairs that were considered the "most similar" would be devices of the same category. Table 3 shows the pairs of systems that were considered the most similar by each measure. The systems are color-coded by category (found in Appendix A).

This was true in almost every case, where the pair of most similar systems was a set of either thermal, wind, or solar harvesters. However, the spectral measure returns different pairs than any of the other measures, finding devices from different categories to be the most similar. In addition, the spectral measure returns groups that have a similarity score of 1 (perfect similarity), despite containing different devices. Even for the other measures, there is no agreement which specific devices were "most similar" in absolute terms (e.g. which was more similar – the pair of wind harvesters or the pair of thermal harvesters?).

TABLE 2. Mean similarity scores of energy harvesting devices grouped by category. Shaded cells indicate within-category means that are lower than the overall mean for the similarity measure.

Measure	Category Mean Similarity Score							Mean Similarity Score
	Inductive (n=9)	*Piezoelectric (n=6)*	*Wind (n=6)*	*Wave (n=3)*	*Solar (n=6)*	*Thermal (n=5)*	*Hybrid (n=4)*	
SMC	0.61	0.64	0.75	0.67	0.54	0.64	0.45	0.49
Jaccard	0.41	0.31	0.58	0.44	0.40	0.52	0.38	0.31
Cosine	0.55	0.44	0.72	0.59	0.55	0.64	0.52	0.44
GED	0.61	0.64	0.75	0.67	0.54	0.64	0.45	0.49
Spectral	0.71	0.70	0.75	0.72	0.61	0.68	0.54	0.65
DeltaCon	0.60	0.67	0.67	0.59	0.54	0.58	0.39	0.47

TABLE 3. Pairs of devices with the highest similarity score (thermal, wind, solar, wave, inductive, and piezoelectric devices). For the spectral measure, there was a group of three devices that had the highest similarity.

Measure	Systems	Similarity
SMC	Micropelt STM-PEM Micropelt TE-power Ring	0.95
Jaccard	Four Seasons Enviro-Energies	0.88
	Tracking System Solar Heat Engine w/ Mirrors	
Cosine	Four Seasons Enviro-Energies	0.93
	Tracking System Solar Heat Engine w/ Mirrors	
GED	Micropelt STM-PEM Micropelt TE-power Ring	0.95
Spectral	Wing Wave Generator Michigan U Piezo Flag	1
	Nova Energy Tuna Turbine WindTamer	
	U Texas Prototype Heel-impact Shoe Harvester	
	Columbia Power Manta Buoy Micropelt STM-PEM Enocean Eco 100	
DeltaCon	Micropelt STM-PEM Micropelt TE-power Ring	0.91

4 Discussion

Due to the fact that functional models can be represented in a mathematical space, they are well suited to a variety of similarity measures beyond those used for text documents and descriptions, and commonly utilized in engineering design. Specifically, there is the possibility to easily represent a functional model as a network to find structural similarities (as was done in this work). However, this work shows that the choice of measure changes empirical findings, and therefore is representative of different interpretations of similarity. Based on previous qualitative analysis on the energy harvesting device data used in this study, it was determined that all of the energy harvesting devices have a similar function structure in general, but differ in some supplemental functions and flows. This overarching structural similarity was not captured in the quantitative metric originally used to compare the devices [13]. Even here, the different measures show different spreads of similarity values as shown in Figure 2. The spectral measure seems to reflect a domain level similarity between the energy harvesting systems through its skew towards higher similarity scores. The spectral distance has been found to work well to distinguish designs when networks have very similar degree matrices but not the same specific functions. In this context, it can be expected that the degree distributions are very similar around common functions, such as *convert* (a key function for energy harvesting devices since they are all converting some input to a form of usable energy flow). On the other hand, measures such as the Jaccard similarity find the complete opposite, indicating low similarity among the set of devices.

4.1 Case Studies

In order to better understand the measures and the types of systems they returned as most similar, some examples are examined in more detail. The measures are qualitatively analyzed under the lens of design exploration and exploitation. Exploration relates to activities that help discover new knowledge while exploitation relates to activities that build on already existing knowledge [39]. In the context of product design, the two lead to different outcomes – exploration can lead to more innovative designs (that may translate to better market performance), while exploitation can lead to a better process performance [40]. Therefore, it can be important to see if measures are more suited to exploration or exploitation.

The measures that have commonalities in their mathematical formulation, such as the cosine and Jaccard similarity or the SMC and GED measures, did not differ significantly in the systems that were found as most similar. Therefore, the measures discussed are ones that were not highly correlated based on the rank coefficient analysis. In general, using a different vector method did not significantly impact the most similar systems. However, using one of the network based methods did change the types of systems identified as the most similar systems. Examples were chosen to compare among both vector and network measures.

Vector Measures

To illustrate the case of a difference among the vector measures, we can examine the group of three devices classified as wave generators (Columbia Power Manta Buoy, Nova Energy Tuna Turbine, and Wing Wave Generator). Using the Wing Wave Generator as an input, we find the most similar device. For all three measures, the result is another wave generator: the Nova Energy Tuna Turbine. This is an expected result, as two wave generators can reasonably work in similar ways. However, using the Columbia Power Manta Buoy as an input, we get more interesting results. For both the Jaccard and cosine similarity, the result is another wave generator: the Nova Energy Tuna Turbine. However, for the SMC, the result is not a wave generator, but a piezoelectric device (Piezo Backpack Straps). Another wave generator does not appear until 4th place and the Nova Energy Tuna Turbine does not appear until 7th place.

The SMC is the only vector measure that finds a device that is not another wave generator as the most similar. As the SMC equally weights functions that exist in the device and functions that do not exist in the device, it is able to find devices similar even if they do not have a lot of functions in common. Specifically, if the devices both have a low number of functions or flows and at least some of them are overlapping, they can still be considered similar. In contrast, the Jaccard coefficient requires that the functions and flows be present in both devices and match. In general, vector measures are more successful at identifying devices that are in the same category (e.g., finding more wave generators if the design idea is a wave generator). Not commenting on the actual usefulness of the out-of-category device returned by the SMC, we can, however, suggest that given only vector options, the Jaccard and cosine measures would likely be better for exploiting, while the SMC might be better for exploring – in this case, suggesting other categories of energy harvesters.

Network Measures

Previous literature on network measures has recommended that the spectral distance is more useful for population comparison (e.g. comparing two different functional models) while the DeltaCon distance is more useful for dynamic comparison (e.g. comparing a functional model that has changed over time for reasons such as failure) [37]. The two network methods, even though they both use the degree matrix, often return different results. To compare the network methods, we can again look at the group of wave generators (Columbia Power Manta Buoy, Nova Energy Tuna Turbine, and Wing Wave Generator). When using the Wing Wave Generator as an input, the DeltaCon distance finds another wave generator (Nova Energy Tuna Turbine) to be the most similar, with a score of 0.75. The spectral distance results in a similarity score of 0.71, yet ranks the same wave generator much lower for similarity (29th), compared to all of the other devices (the high similarity score and low ranking reflects the skew in the distribution of spectral similarity as shown in Figure 2). The Wing Wave Generator is most similar to a wind device (Michigan U Piezo Flag) using the spectral distance and another wave device (Nova Energy Tuna Turbine) using the DeltaCon distance and GED. This example suggests that the spectral distance has results that are more unexpected and might lend itself better to design exploration, though they may be less interpretable.

4.2 Implications for Design-by-Analogy

If analogical stimuli are provided to designers computationally, the choice of similarity measure to retrieve the "right" stimulus becomes important. Work in analogical design has implied that stimuli from a "sweet spot" between near-field and far-field help designers in the design process, but has also noted that the meaning of near and far varies across the literature [5]. The results of this work indicate that the choice of similarity measure impacts what types of systems are considered functionally similar (and consequently, what might be returned as near or far). For instance, measures that are better able to capture structural similarity might return, as a near example, a result that another measure (or a human designer) might consider a far example.

More broadly, some measures might be better suited to either design exploitation or design exploration. A higher-level notion of similarity can be useful to provide unintuitive but similar examples that aid divergence during design exploration. However, it may not be useful when designers want to quickly transfer aspects of an existing design that matches their needs to a new design. In the latter case, which can be important when converging on an idea later in the design process, it might be better to have a measure that returns designs that are more intuitively similar (e.g. for energy harvesters, a within category device). Therefore, these results can lead to a better understanding of how specific similarity measures can be leveraged for specific purposes within engineering design.

For near (within domain) systems, the choice of similarity measure is not as critical – most of the measures tend to result in the same sets of systems returned as "similar." However, if the functional model is represented as a network, very different results can be found using the spectral distance. Therefore, a network measure like the spectral measure may not perform particularly well to exploit already refined target designs, but could be more useful than vector-based methods for design exploration.

A within-domain data set was useful for initial work because the designs had an aspect of known similarity. However, there was variability present even though the group had similar working principles. The question of how each similarity measure would handle out-of-domain systems is still unanswered. In addition, mapping the functional models with no functions repeated, or no weighting of importance, is unlikely to work for a very complex system that has most or all of the functions from the functional basis. In this case, we refer to increasing complexity as having an increasing number of function-flow pairs. A more complex system can be mapped to a network as demonstrated in previous work [29, 30] if more detail about function repetition or importance is available. Even at the current level of detail, the results indicate that the complexity of a system influences the output of design similarity for each measure. Therefore, the choice of similarity measure might depend on whether the desired task is for design exploitation or exploration, as well as depend on the types of systems in question. Although further research is needed, we propose that the similarity measures can be interpreted as follows in the context of functional design representations:

Simple matching coefficient (SMC). This measure can utilize the *absence* of key function-flow pairs in systems as information of their similarity. For this reason, it is highly coupled with system complexity. The measure may work better for design exploitation in more complex systems, where the absence of specific functions or flows is meaningful. However, it may return more unexpected results between less complex systems that have some shared aspects, but also many function-flow pairs missing.

Jaccard similarity coefficient. The shared *absence* of function-flow pairs between systems does not increase the similarity. Only function-flow pairs that exist in at least one of the systems and their positions contribute to the definition of similarity. The measure would likely then be useful for design exploitation in both high and low complexity systems.

Cosine similarity. Similar to the Jaccard measure, function-flow pairs that exist and match define cosine similarity, making it useful for design exploitation in both high and low complexity systems.

Graph edit distance (GED). This measure acts as a version of the SMC for when the functions and flows are represented as a network.

Spectral distance. Similarity depends on how the functions are connected to flows (not on what the specific functions and flows actually are). Systems where function-flow chains have the same structure will be considered similar. The ability for the spectral measure to convey a notion of "structural similarity" gives it a higher potential for design exploration. This measure is the only measure that can find different systems to be exactly the same, and as such would not likely be used for design exploitation. It does not seem particularly affected by the system's complexity.

DeltaCon distance. Similarity depends primarily on the matching of specific function-flow pairs (as in GED). However, the structure of how the functions are connected to flows has an influence on similarity particularly as a function has many flows connected to it or vice versa. Therefore, if a system is more complex, the influence of the "structural similarity" aspect is more visible making the measure more suitable for design exploration. Otherwise, the results are similar to GED.

Given these interpretations, potential directions for choosing a similarity measure based on the application and system complexity are shown qualitatively in Figure 5.

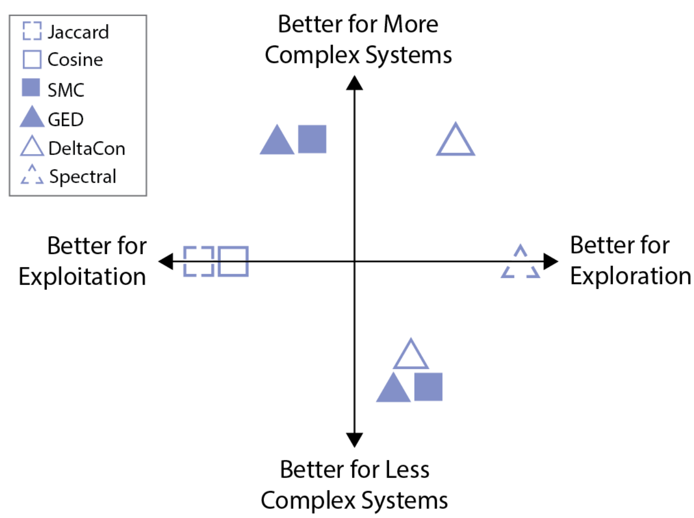

FIGURE 5. Measures are shown based on proposed use scenarios. Some measures are better suited for exploitation or exploration depending on the complexity of the systems being considered, and therefore may be shown multiple times depending on the application.

The energy harvesters represent a set of systems of varying complexity that might not have surface or form similarities, but are related to each other functionally. In the case of utilizing design-by-analogy for such systems, a computational approach to finding the similarity between them might be particularly useful. However, since it is possible to define the similarity between systems in several ways, the measure choice can lead to different analogies that might influence a designer's trajectory. Therefore, to truly understand the use of specific similarity measures in the context of design exploration or exploitation, it would also be important to determine whether the types of results provided by unexplored network-based distances, like the spectral distance, would be useful to designers in practice. Would the measures be retrieving examples that are "too far" or just right?

5 Conclusion

An empirical analysis of different similarity measures to determine the similarity of functional models (mapped as a vector or a network) indicated that the choice of measure can significantly affect which designs were returned as similar to a target design. The use of a set of functional models from within the same technological domain suggests that the different measures captured varying aspects of similarity. The analysis found network measures to be a potentially viable alternative to vector measures, depending on the design context; this is particularly relevant to determining near vs. far analogical stimuli and aiding in design exploitation vs. exploration. This work is a step toward understanding which similarity measures should be used in different design relevant contexts. Though only tested on functional models in the present study, the results imply the need to carefully consider the choice of similarity metric in research that requires a measurement of design similarity, regardless of the design representation.

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APPENDIX A Categorization of Energy Harvesting Devices

Category	Systems
Inductive	Perpetuum FSH/C
	Enocean Eco 100
	Clarkson U Prototype
	Michigan U PFIG
	U Texas Prototype
	Seiko Kinetic Watch
	AA Battery Harvester
	Socket
	Kinetic Flashlight
Piezoelectric	MIDE Volture
	Bistable Buckling Harvester
	Heel-impact Shoe Harvester
	Innowattech Road/Rail
	Piezo Backpack Straps
	U Texas Prototype
Wind	WindTamer
	Leviathan
	Enviro Energies
	Four Seasons
	Humdinger Wind Belt
	Michigan U Piezo Flag
Ocean-current/Wave	Nova Energy Tuna Turbine
	Columbia Power Manta Buoy
	Wing Wave Generator
Solar	Solar Heat Engine w/ Mirrors
	Tracking System
	Inflatable Mat
	Big Belly Trash Compactor
	Transparent Film on Window
	Seiko Solar Watch
Thermal	Seiko Thermic Watch
	Enocean ECT 310 Perpetuum
	Micropelt TE-power Probe
	Micropelt TE-Power Ring
	Micropelt STM-PEM
Hybrid	Solar Powered Sterling Engine
	Solar/Wind Streetlamp
	Kinesis Wind/Solar
	Hymini Wind/Solar Crank

APPENDIX B List of Functions and Flows

Functions	Flows
Separate	Solid
Distribute	Human
Import	Gas
Export	Liquid
Transfer	Human Energy
Guide	Mechanical Energy
Couple	- *Rotational Mechanical Energy*
Mix	- *Translational Mechanical Energy*
Actuate	- *Vibrational Mechanical Energy*
Regulate	Pneumatic Energy
Change	Hydraulic Energy
Stop	Light Energy
Convert	Electrical Energy
Store	Magnetic Energy
Supply	Thermal Energy
Sense	Status
Indicate	Control
Process	
Stabilize	
Secure	
Position	